Problem 1. (15 points)

You need to solve the following set of simultaneous linear algebraic equations:

\[ Ax = c \]

Write a MATLAB script that will solve this set for the vector \( x \). You must use MATLAB language, MATLAB commands, and/or MATLAB programs developed in your text.

This program must be complete with input statements for entering the matrix \( A \) and the vector \( c \), and display of the result vector \( x \). Your program must perform the solution only if matrix \( A \) is nonsingular. If \( A \) is singular, then the program must display the message: 'Matrix A is singular of rank = ', show the rank of \( A \), and quit.

The program must be syntactically correct, i.e. it should execute correctly in the MATLAB command window.

Problem 2. (25 points)

Derive the equation for the second order derivative of \( y \) in terms of central differences with error of order \( h^6 \). Your final solution should have like-terms grouped together.

Problem 3. (30 points)

Calculate a solution to the following set of linear algebraic equations:

\[
\begin{align*}
    x_1 + 2x_2 + 3x_3 &= 4 \\
    4x_1 + 5x_2 + 6x_3 &= 5 \\
    7x_1 + 8x_2 + 9x_3 &= 6
\end{align*}
\]

(a) Use the Gauss elimination method. The accuracy of your solution is important. Show each step clearly.

(b) Once you have obtained the answer, verify that it satisfies all three of the equations. Show this clearly.
Problem 4. (30 points)

The Underwood equation for multicomponent distillation is given as

$$\left(\sum_{j=1}^{n} \frac{\alpha_j z_j F}{\alpha_j - \phi}\right) - F (1 - q) = 0$$

where

- $F$ = molar feed flow rate
- $n$ = number of components in the feed
- $z_j F$ = mole fraction of each component in the feed
- $q$ = quality of the feed
- $\alpha_j$ = relative volatility of each component at average column conditions
- $\phi$ = root of the equation

It has been shown by Underwood that $(n - 1)$ of the roots of this equation lie between the values of the relative volatilities as shown below:

$$\alpha_n < \phi_{n-1} < \alpha_{n-1} < \phi_{n-2} < ... < \alpha_3 < \phi_2 < \alpha_2 < \phi_1 < \alpha_1$$

In other words, for the three-component mixture given in Table 1, the Underwood equation will have two roots located as shown here:

$$1.00 < \phi_2 < 2.00 < \phi_1 < 5.00$$

For the values given in Table 1, evaluate the two roots using the Newton-Raphson method. Perform three iterations for each root.

Table 1

<table>
<thead>
<tr>
<th>Component in feed</th>
<th>Mole fraction, $z_j F$</th>
<th>Relative volatility, $\alpha_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.40</td>
<td>5.00</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.30</td>
<td>2.00</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.30</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$F = 100 \text{ mol/h}$  \quad  q = 1.0 \text{ (saturated liquid)}