Problem set 9 ADV. TRANSPORT II 2012
Due by April 10, 05:00 pm. Please, keep a copy for corrections against the solutions that will be posted April 11.
Corrected homework set is due April 17.

Problem 1 (BSL 10B.3)

Heat conduction in a nuclear fuel rod assembly (Fig. 10B.3). Consider a long cylindrical nuclear fuel rod, surrounded by an annular layer of aluminum cladding. Within the fuel rod heat is produced by fission; this heat source depends on position approximately as

\[ S_n = S_\text{cl} \left[ 1 + p \left( \frac{r}{R_c} \right)^2 \right] \]  

(10B.3-1)

Here \( S_\text{cl} \) and \( p \) are known constants, and \( r \) is the radial coordinate measured from the axis of the cylindrical fuel rod. Calculate the maximum temperature in the fuel rod if the outer surface of the cladding is in contact with a liquid coolant at temperature \( T_C \). The heat transfer coefficient at the cladding-coolant interface is \( h_C \), and the thermal conductivities of the fuel rod and cladding are \( k_f \) and \( k_C \).

**Answer:**

\[ T_{f,\text{max}} - T_l = \frac{S_\text{cl} R_C^2}{4k_f} \left( 1 + \frac{b}{4} \right) + \frac{S_\text{cl} R_C^2}{2k_C} \left( 1 + \frac{p}{2} \left( \frac{k_f}{k_C} \ln \frac{R_c}{R_f} \right) \right) \]

Problem 2 (BSL 10B.4)

Heat conduction in an annulus (Fig. 10B.4).

(a) Heat is flowing through an annular wall of inside radius \( r_0 \) and outside radius \( r_1 \). The thermal conductivity varies linearly with temperature from \( k_0 \) at \( T_0 \) to \( k_1 \) at \( T_1 \). Develop an expression for the heat flow through the wall.

(b) Show how the expression in (a) can be simplified when \( (r_1 - r_0)/r_0 \) is very small. Interpret the result physically.

**Answer:**

(a) \( Q = 2\pi L (T_0 - T_1) \left( \frac{k_0 + k_1}{2} \right) \left( \ln \frac{r_1}{r_0} \right)^{-1} \)

(b) \( Q = 2\pi r_0 L \left( \frac{k_0 + k_1}{2} \right) \left( \frac{T_0 - T_1}{r_1 - r_0} \right) \)

Fig. 10B.4. Temperature profile in an annular wall.
Problem 3 (BSL 10B.8)

Electrical heating of a pipe (Fig. 10B.8). In the manufacture of glass-coated steel pipes, it is common practice first to heat the pipe to the melting range of glass and then to contact the hot pipe surface with glass granules. These granules melt and wet the pipe surface to form a tightly adhering nonporous coat. In one method of preheating the pipe, an electric current is passed along the pipe, with the result that the pipe is heated (as in §10.2). For the purpose of this problem make the following assumptions:

(i) The electrical conductivity of the pipe $k_e$ is constant over the temperature range of interest. The local rate of electrical heat production $S_e$ is then uniform throughout the pipe wall.

(ii) The top and bottom of the pipe are capped in such a way that heat losses through them are negligible.

(iii) Heat loss from the outer surface of the pipe to the surroundings is given by Newton’s law of cooling: $q = h(T_i - T_o)$. Here $h$ is a suitable heat transfer coefficient.

How much electrical power is needed to maintain the inner pipe surface at some desired temperature, $T_o$, for known $k, T_o, h$, and pipe dimensions?

\[ P = \frac{\pi R^2(1 - \kappa^2)L(T_o - T_i)}{\frac{(1 - \kappa^2)R}{2h} - \frac{(\kappa R)^2}{4k} \left[ \frac{1}{\kappa^2} - 2\ln \kappa \right]} \]

Fig. 10B.8. Electrical heating of a pipe.