Problem 1:
From Example 8.1 or notes, with $z$ equals to $L$

$$\frac{c_1}{c_1(sat)} = 1 - e^{-k a L/v_0}$$

Please notice that the conditions given are “liter/min” for flow rate and “m$^2$” for area.

So $k = \frac{\text{volumetric flow rate}}{\text{area}} \ln \left(1 - \frac{c_1}{c_1(sat)}\right)$, with the unit [cm/s]

(Compare this equation with the equation on p.241)

(a) $k = \frac{1800/60}{12500} \ln(1-\frac{2 \times 10^{-4}}{2.675 \times 10^{-2}}) = 3.3 \times 10^{-3}$ [cm/s]

(b) $l = \frac{D}{k} = 1.8 \times 10^{-5}/3.3 \times 10^{-3} = 5.45 \times 10^{-3}$ [cm]

(c) $t = \frac{4D}{\pi k^2} = 2.1$ [s]

(d) $\tau = \frac{D}{k^2} = 1.8 \times 10^{-5}/(3.3 \times 10^{-3})^2 = 1.65$ [s]

Problem 2:
In the ether phase, $k_e = \frac{D}{l} = 10^{-5}/10^{-2} = 10^{-3}$ [cm/s]

In the water phase, $k_w = \sqrt{D/\tau} = \sqrt{10^{-5}/10} = 10^{-3}$ [cm/s]

Partition coefficient H is 200

$\therefore K_e = (1/k_w + 1/Hk_e)^{-1} = 9.95 \times 10^{-4}$ [cm/s]

In the ether phase,

$$N(\text{flux})A = V \frac{dc_e}{dt} = V \frac{c_e - c_{e0}}{t - 0}$$

$$c_e = c_{e0} + \frac{KAt}{V} \left(c_{wo} - \frac{c_{e0}}{H}\right) = 5 \times 10^{-3} + \frac{9.95 \times 10^{-4} \cdot 12 \cdot 3600 \left(5 \times 10^{-3} - 5 \times 10^{-3}\right)}{20}$$

$$= 1.57 \times 10^{-2} [M]$$

Problem 3:

$$k = 0.276 \frac{D_p}{R_T a} \left(Re^{1/2} Sc^{1/3}\right) = 0.276 \cdot \frac{0.08}{62.05 \times 298^2} \cdot 1000 = 4.52 \times 10^{-4}$$ [cm/s]

$\therefore l = \frac{D}{k} = 177$ [cm]

$\therefore \tau = \frac{D}{k^2} = 391574$ [sec] is about 109 hours

Problem 4:

$$\frac{dV}{dt} \ (\text{cm}^3/\text{day}) = 0.02 \ p \ (\text{mm Hg}) \ \text{M} \propto pM$$

means the rate of diffusion is proportional to pressure and egg mass

Thus, $\frac{1}{p} \frac{dV}{dt} \ (\text{cm}^3/\text{day-mm Hg}) = 0.02 \ M$

The diffusion occurs through the pore channels cross-section area that correlates with the egg mass, as
A \ (\text{mm}^2) = 0.04 \ \text{M}

The diffusion length is the length of the pore channel that correlates with the egg mass, as

\( l \ \text{(mm)} = 0.03 \sqrt{\text{M}} \)

From the film theory,

\[
N_1 = \frac{D\Delta c}{l} = D \frac{A}{l \ \text{RT}} = \frac{D}{RT \ 0.03\sqrt{\text{M}}} \rho \propto p \sqrt{\text{M}}
\]

This means that the rate of diffusion should be proportional to the pressure and the square root of egg mass.

Problem 5

The mass transfer equation of this problem is

\[
\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2}
\]

The boundary conditions and initial condition are:

- IC \quad c(z, 0) = 0
- BC1 \quad c(0, t) = c_0
- BC2 \quad c(2b, t) = c_0

*You can use your own initial condition and boundary conditions based on the coordinate you choose, so as the following dimensionless parameters. However, they should be consistent to the answer. Using dimensionless parameters, we can make the governing equation simpler.

\[
\theta = \frac{c - c_0}{c_0}, \quad \zeta = \frac{z}{b} \quad \text{and} \quad \tau = \frac{tD}{b^2}
\]

The governing equation and the corresponding IC and BCs become:

\[
\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \zeta^2}
\]

- IC \quad \theta(\zeta, 0) = 0
- BC1 \quad \theta(0, \tau) = 1
- BC2 \quad \theta(2, \tau) = 1

To solve this PDE, you will need to use the separation of variables.

*If you were using the Zill’s *Advanced Engineering Mathematics* as the textbook in the math class, it is in the section 13.3.

\[
\theta(\tau, \zeta) = Z(\zeta)T(\tau)
\]

\[
\frac{T'}{T} = \frac{Z''}{Z} = -\alpha^2
\]
$T' + \alpha^2 T = 0$ can be solved first, and $T = Ae^{-\alpha^2 \tau}$

*For the purpose of simplicity, arbitrary coefficients “A” and B will keep being replaced.

$Z' + \alpha^2 Z = 0$

The solution is $Z = A \sin(\alpha \zeta) + B \cos(\alpha \zeta)$

Because $Z(0) = 0$ and $Z(2) = 0$, we can solve that $B = 0$ and $\alpha = \frac{n \pi}{2} \cdot n = 1, 2, \ldots \infty$

$$\theta(\tau, \zeta) = Z(\zeta)T(\tau) = \sum_{n=1}^{\infty} A_n e^{-\frac{n^2 \pi^2}{4} \tau} \sin \left(\frac{n \pi}{2} \zeta\right)$$

To satisfy the IC,

$$\sum_{n=1}^{\infty} A_n \sin \left(\frac{n \pi}{2} \zeta\right) = 1$$

To solve coefficient $A_n$, we need to multiply $\sin \left(\frac{n \pi}{2} \zeta\right)$ and integrate the equation from 0 to 1. We will have $A_n = \frac{2}{2} \int_0^1 \sin \left(\frac{n \pi}{2} \zeta\right) d\zeta = \frac{2}{n \pi} \left(1 - (-1)^n\right)$

The final solution will be

$$\theta(\tau, \zeta) = Z(\zeta)T(\tau) = \sum_{n=1}^{\infty} \left(\frac{2}{n \pi} (1 - (-1)^n)\right) e^{-\frac{n^2 \pi^2}{4} \tau} \sin \left(\frac{n \pi}{2} \zeta\right)$$

Switch back to the original parameters:

$$\frac{c_0 - c}{c_0} = \sum_{n=1}^{\infty} \left(\frac{2}{n \pi} (1 - (-1)^n)\right) e^{-\frac{n^2 \pi^2 t D}{4 b^2}} \sin \left(\frac{n \pi z}{2 b}\right)$$

$$c = c_0 \left\{1 - \sum_{n=1}^{\infty} \left(\frac{2}{n \pi} (1 - (-1)^n)\right) e^{-\frac{n^2 \pi^2 t D}{4 b^2}} \sin \left(\frac{n \pi z}{2 b}\right)\right\}$$