Granular mixing is a vital operation in food, chemical, and pharmaceutical industries. Although the tumbling blender is by far the most common device used to mix grains, surprisingly little is known about mixing or segregation in these devices. In this paper, we report the first fully three-dimensional particle dynamics simulations of granular dynamics in two standard industrial tumbling blender geometries: the double-cone and the V-blender. Simulations for both monodisperse and bidisperse (segregating) grain sizes are performed and compared with experiment. Mixing and transport patterns are studied, and we find in both tumblers that the dominant mixing mechanism, azimuthal convection, contends against the dominant bottleneck, axial dispersion. The dynamics of blending, on the other hand, differs dramatically between the two tumblers: flow in the double-cone is nearly continuous and steady, while flow in the V-blender is intermittent and consists of two very distinct processes.

Keywords: Granular, Mixing, Simulation, Tumblers

1. Introduction

The importance of granular mixing to the US economy is immense. Goods ranging from pharmaceuticals and polymers to semiconductors and ceramics increasingly depend on highly reliable granular flow and highly uniform granular mixing. Specific examples are easily found. The energy reserves of coal in the United States exceed the combined oil reserves in the rest of the world, yet due to processing limitations, in excess of 2 billion tons of coal fines are currently stored as hazardous wastes in the continental US [Killmeyer, 1993]. Likewise, the annual cost of inefficient industrial mixing in the US has been estimated to be as high as $10 billion, with roughly 60% of blended products typically being powdered or granular [Nienow, 1992].

The most common batch mixers in industrial use are tumbling blenders, where grains are caused to flow by a combination of the action of gravity and the rotating motion of the mixer. Two of the most common industrial tumbling blenders are the double-cone and the V-blender (depicted in Fig. 1), which together comprise a high fraction of all batch blending applications. For a review on solid mixing devices, see [Fan 1972; Fan 1990; Carley-Macauley 1962, 1964] and references therein.

The flow pattern within these mixers is believed to consist of a thin, rapid flow region near the surface, a nearly non-deforming region beneath that rotates with the container as a solid body, and a narrow transition region between, which is characterized by high shear and density gradients. This picture, however, is best understood in the context of quasi-two-dimensional disk or drum blenders [Hogg 1972; Khakhar 1997]. The analytic foundation for the understanding of flow patterns in fully three-dimensional tumblers such as the double-cone or V-blender is much weaker, and indeed the work that we discuss here
indicates that this quasi-2D picture does not carry over well into truly three-dimensional tumblers. To date, the design and control of realistic, three-dimensional blenders has been based more on trial and error than on quantitative or analytic methods. Even quantitative characterizations of mixing performance as a function of the most basic parameters, such as vessel speed or filling level, are scarce in the literature [Brone 1998a; Brone 1997a; Muzzio 1997; Brone 1998b].

In this paper, we apply particle dynamics techniques to study flow and mixing in double-cones and V-blenders. While this approach is currently limited in particle numbers to $O(10^4)$, it is deemed adequate for freely flowing materials, which are commonly present in industries ranging from fertilizers and foods to alloys and plastics, and which often present profound mixing challenges due to strong segregational tendencies. Cohesive materials, which exhibit different phenomenology, usually require high shear to mix homogeneously and are often processed in other types of equipment.

The plan for this paper is as follows. A brief review of granular flow models is presented in §2, and the computational method used is summarized in §3. Numerical results are presented and compared with experiment in §4 for the double-cone tumbler and in §5 for the V-blender. Finally, brief conclusions are presented in §6.

## 2. Granular flow modeling

Models of granular flows can be broadly divided into three categories: continuum, kinetic theory, and discrete.

Continuum models neglect the discrete nature of grains and assume a continuous variation of matter that obeys conservation laws of mass and momentum. The behavior of the material is thus assumed to be described by constitutive equations, frequently \textit{ad hoc}, that relate kinematic, mechanical, and thermal field variables. Examples of continuum models are those adapted from soil plasticity [e.g. Spencer 1982] and fluid mechanics [e.g. Goodman 1972]. Some continuum models incorporate microstructural parameters that have the ability to describe characteristics and mechanisms peculiar to granular materials, such as the solid fraction distribution function and dilatancy phenomena [e.g. Mehta 1990]. While many of these models are limited to high speed flows where either the Coulombic friction or collisional interactions dominate, models that incorporate both types of behavior have also been proposed [e.g. Sayed 1981; Johnson 1987; McTigue 1987].

Kinetic theory models exploit similarities between interacting grains and colliding molecules in a dense gas [Chapman 1970]. These models incorporate energy dissipation that is characteristic of granular flows and can be applied in certain circumstances, such as at the surface of an agitated granular mass [Esipov 1997]. The fluctuational velocity distribution is in that case taken to be approximately Maxwellian, and continuum hydrodynamic forms of conservation laws of mass, momentum, and fluctuational kinetic energy (temperature) are then established by averaging methods [Savage 1981; Jenkins 1985].

Discrete models admit numerous subclassifications, but all take the constituent grains to be distinct and to move according to prescribed rules. Examples of discrete models include Monte Carlo methods [Rosato 1986; Barker 1990; Hopkins 1992], which apply probabilistic rules; cellular automata [Baxter 1991; Shinbrot 1997a], which use deterministic (and often simplified) rules; and particle dynamics, whose rules are derived from first principles. The use of the latter methods for granular flows is supported by the success of molecular dynamic simulations for gas and liquid systems. Furthermore, with
the increase in computational power, the use of particle dynamics techniques has proliferated in recent years [e.g. Walton 1986; McNamara 1992; Goldhirsch 1993; Luding 1994a; Constantin 1995; Gallas 1996; Ristow 1996; Muguruma 1997; Bizon 1998].

Two types of particle dynamics methods are most common: 'hard-particle' methods, in which collisions are instantaneous and binary, and 'soft-particle' methods, in which collisions can be lasting and multiple. Hybrid [Hopkins 1991; Louge 1994; McCarthy 1998] and mean-field [Shinbrot 1997b] methods have also been proposed. Typically, hard-particle methods are most useful in rapid granular flows, where collisions are discrete and distinct. In this case, one determines the outcome of each individual collision and evolves particle trajectories ballistically between collisions. Computational load is occupied in this case by re-ordering future events (collisions) following each collision. Soft-particle methods, by contrast, are used in situations where contacts are enduring, rather than distinct. In this situation, particles are permitted to suffer minute deformations, and these deformations are used to compute restoring elastic, plastic, and frictional forces. Of necessity, particle contacts persist in tumbling blenders, so we adopt the soft-particle approach here.

3. Soft-particle method

The soft-particle method was developed by Cundall [Cundall 1971; Cundall 1979], and has been used to simulate chute flow [Dippel 1996], heap formation [Luding 1997], hopper discharge [Thomson 1991; Zhang 1992; Ristow 1994a], vibrated beds [Gallas 1992; Luding 1996; Bizon 1998], and flows in rotating drums [Ristow 1996; Khakhar 1997; Wightman 1997]. Several excellent reviews are available, for example Mehta [1994] Ristow [1994b], Schäfer [1996] and Dippel [1998].

In the present study, the granular material is idealized as a collection of frictional and inelastic spherical particles. Each particle may interact with its neighbors or with the boundary of the tumbler through both normal and tangential forces. The elastic modulus and computational timestep are chosen so that deformations of particles remain small when compared with their displacements and diameters [cf. Luding 1994b].

The normal forces that develop between particles in contact are calculated using the "partially latching spring" model of Walton and Braun [1986]. This model approximates the elastic-plastic behavior of contacting spheres observed in laboratory experiments [Goldsmith 1964; Hopkins 1991] and in finite element calculations [Walton 1993b]. The normal force \( F_{ij}^n \) acting on particle \( i \), with position vector \( \mathbf{r}_i \) and radius \( R_i \), resulting from its interaction with particle \( j \), with position vector \( \mathbf{r}_j \) and radius \( R_j \), is taken to be the following function of the overlap \( \alpha_{ij} = |\mathbf{r}_i - \mathbf{r}_j| - (R_i + R_j) \) for loading and unloading respectively:

\[
F_{ij}^n = \begin{cases} 
K_1 \alpha_{ij} n_{ij}, & \hat{\alpha}_{ij} \geq 0 \ (\text{loading}) \\
K_2 (\alpha_{ij} - \alpha_o) n_{ij}, & \hat{\alpha}_{ij} \leq 0 \ (\text{unloading})
\end{cases}
\]

(1)

where \( n_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j| \) is the unit vector joining the centers of the two particles, and \( K_1 \) and \( K_2 \) are the normal stiffness coefficients for loading and unloading, respectively. The stiffness coefficient for unloading is taken to be larger than that of loading, after Walton [1986]. For simple, binary collisions, this model produces a

\[ \text{In our simulations, we use } K_1 = 3000 \, \text{N/m}, \quad K_2 = K_1/\varepsilon^2, \quad \varepsilon = 0.6 \]
coefficient of restitution, $\varepsilon = \sqrt{K_1/K_2}$, that is independent of the relative velocity of impact [Schäfer 1996].

Tangential forces are calculated using Walton's [1993a] three-dimensional "incrementally slipping" model. This model is based on Mindlin's [1949] theoretical work on frictional contact. After contact occurs, static frictional force builds up nonlinearly with displacements in the tangent plane of contact. Once a threshold of maximum static frictional force is reached, bodies slide with constant dynamic friction coefficient.

We track particle interactions using linked list algorithms described elsewhere [e.g. Grest 1989], and integrate the resulting equations of motion using a 'leap-frog' algorithm [Allen 1987]. The equations of motion are written as a system of two sets of ordinary differential equations: one for positions and the other for velocities. The latter is integrated first and then the former. After this, the accelerations (forces) are evaluated for the next time step.

Particles in a tumbling blender can be in relative motion with respect to the blender's boundary, which in turn rotates with respect to a fixed frame. To simulate particle motions within vessels undergoing a general rotational motion (not necessarily a simple rotation about a fixed axis), it is useful to write the equations of motion in a frame attached to the vessel. The advantage of this choice is that contact detection between particles and the blender's inner wall, and computation of the resulting particle-wall forces, are considerably simplified.

![Figure 1 - geometries of double cone & V-blender](image)

For the containers in our simulations, we use smooth but frictional boundaries, defined as follows. Referring to Fig. 1, for the double-cone we use two sizes. In monodisperse simulations, we use a tumbler with cone angle $A = 45^\circ$, diameter $2R = 7.62$ cm, cone height $D = 3.24$ cm, and cylinder height $2L = 2.67$ cm, while in bidisperse simulations we use a tumbler with cone angle $A = 45^\circ$, diameter $2R = 25.4$ cm, cone height $D = 10.795$ cm, and cylinder height $2L = 8.9$ cm. For the V-blender, we compose two identical cylindrical shells joined at an angle $a = 90^\circ$, where each shell has diameter $d = 7.62$ cm, maximum length $l = 11.43$ cm, and distance perpendicular to the axis of rotation from the axis to the intersection between shell central axes $s = 2.69$ cm. Our experimental containers

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2 The advantage of using the leap-frog integrating scheme is that it is second order accurate, explicit, simple, and does not require the storage of auxiliary variables. Predictor-corrector integrating schemes by contrast can provide more accurate solution to the equations of motion if at least two or three correction steps are used. This requires expensive force evaluations, and consequently these schemes are rarely used in molecular and particle dynamics simulations [Allen 1987]. This is also true for higher order methods such as Runga-Kutta algorithms.
are made of Plexiglas™ and are of identical geometry, with the exceptions that the physical double-cone ends are rounded, and the physical V-blender arms meet at an angle $\alpha = 84^\circ$.

4. Case 1: Mixing and segregation in the double-cone blender

Mixing Mechanism:

Prior experimental studies [Brone 1997, 1998a] of mixing and segregation in the double-cone have laid the groundwork for this research. The qualitative picture painted by these studies is that blenders with a reflectional symmetry suffer impeded transport across the symmetry plane. This picture is reinforced and quantified in our particle dynamics study. A synopsis\(^3\) of our results appears in Fig. 2. Here we follow 15,000 identical spheres of diameter 2 mm (fill 44% of total volume) through 6 revolutions of the blender at 15 rpm. Particles are initially loaded from above in random, non-overlapping positions and are allowed to fall to rest before the simulation is begun. After one revolution, we sketch the approximate boundary of the free surface with a dashed line in the figure. Particles initially in the upper right quadrant are colored yellow, and roughly speaking it is observed that the yellow particles travel from top to bottom once per cycle as material is turned over in the tumbler. This is an effect of the level of fill (to be discussed in future papers): a less-filled tumbler would turn its contents over more times in a single revolution that a more-filled tumbler.

Nevertheless, the simulations reveal the essential convective motion that one would naively expect to find in a tumbling blender. Beyond this, Fig. 2 shows that after 6 revolutions, the yellow particles appear to be uniformly dispersed within the right half of the tumbler, but very little transport is evident between the two halves of the blender. For comparison, we show snapshots of an experiment in which identical but colored 1.6 mm spherical glass beads are tumbled in a double-cone at 16 rpm for 6 revolutions. In both simulation and experiment, particle motion evidently consists of an azimuthal convective flow accompanied by a distinct axial diffusive transport, with a significant bottleneck to mixing across the symmetry plane of the tumbler. We quantify these observations next.

![Figure 2 - Top: time sequence of front view from particle dynamic simulations of monodisperse particles differing only by color in double-cone blender. N represents the number of revolutions experienced by the tumbler. The top of the tumbler is rotated frontward, and the free surface is outlined by a dashed line at N = 1. Bottom: corresponding experimental snapshots for nearly identical initial and operating conditions in a transparent tumbler. The tumbler is stopped at the indicated time for each snapshot and is emptied and re-loaded for subsequent snapshots.](http://sol.rutgers.edu/~shinbrot/Moakher/DC.mpg)

\(^3\) For simulation, see http://sol.rutgers.edu/~shinbrot/Moakher/DC.mpg.
In Figure 3, we display the transport of initially vertical colored stripes of spheres. The loading and subsequent motion of particles are as before; we have merely applied different colors to particles whose centers pass through specified regions of space. At $N = 0.5$ we again outline the free surface with a dashed line.

![Figure 3 - Time sequence of initially vertical colored stripes of particles in the same double-cone simulation shown in Fig. 2. Mixing is comparatively rapid within either half of the tumbler, but mixing across the symmetry plane is slow.](image)

Quantification of Mixing Rate:

By tracking the axial coordinates of particles initially within each vertical stripe, we can evaluate the axial mixing flux in the double-cone, as shown in Fig. 4. In the three upper insets, we plot the concentrations of the outermost (light gray), central (gray) and innermost (black) particles as a function of axial position after 1, 3 and 6 revolutions of the tumbler. The colored horizontal lines in the upper insets correspond to uniform concentrations\(^4\). In the main plot, we display the logarithm of the variance of concentration as a function of the number of tumbler revolutions for each of these three sets of particles.

From the main plot of Fig. 4, we note that the upper curve -- corresponding to particles initially near the axial center of the tumbler -- has a nearly exponential scaling region, extending from about 0.5 to 6 revolutions. The lower curve -- corresponding to particles initially near the axial extremes -- is more curved, indicating that the mixing is nearly exponential -- i.e. diffusive [Brone 1998b] -- near the axial center of the blender, but is more complex toward the axial extremes. This fits with the qualitative picture mentioned earlier: mixing within either half of the blender occurs through a combination of convection and diffusion, while mixing between halves is chiefly diffusive. Likewise the upper insets to Fig. 4 show the evolution of concentrations of particles initially in the colored vertical stripes and reflect the same behavior. Practically speaking, the implication is clear: breaking the symmetry of the blender is key to accelerating mixing [cf. Brone 1997].

\(^4\) Each of these three regions contains a different number of particles, hence concentrations are normalized differently within each region. For this reason, the indicated concentration corresponding to a uniform blend differs in each inset.
Velocity Fields:

A great advantage of computational models is that one can easily extract useful results that would be difficult to obtain experimentally. In Fig. 5, we display the mean velocity field for the double-cone at 8 phases of the rotation cycle. Each velocity vector is formed by averaging over both space and time: over space, the velocities of particles lying within small cubic volume elements are averaged, and over time, velocities of particles at the phase of rotation indicated are averaged over the final 4 cycles of tumbler motion. Apparently at each phase there is a distinct flowing layer near the surface and region beneath that moves slowly as a single unit. This is in keeping with the current understanding of tumbler flows described in §2, however this is not always the case in 3D tumblers, as will be shown shortly.

A top view of the velocity field (right of Fig. 5) shows two additional effects: first, there is a sizeable gradient in velocities, with the highest velocities near the center of the flowing layer, and second the velocity field curves to follow the profile of the container sides. This curvature leads to significant mixing and segregation effects. This is another facet of the phenomenon described in Fig. 4, indicating that convective axial mixing is enhanced further from the axial center of the tumbler. Moreover, we can infer segregational implications from this velocity field: large, fast moving, spheres should travel more easily along straight paths than smaller spheres. Correspondingly, the velocity field shown in Fig. 5 suggests that for polydisperse mixtures in the flow regime studied here, larger particles may migrate toward the high speed rectilinear flow region in the center of the tumbler, leaving smaller particles to migrate to follow the slower, curved pathlines near the axial extremes.
Segregation in the Double-cone:

This suggestion is only a heuristic, yet it appears to be followed in this system in the flow regime studied. In Fig. 6(a), we display a top view of a simulation\(^5\) using a bidisperse mixture of approximately equal masses of large and small beads. Here we have used 11,500 small (6 mm diameter, blue) spheres initially randomly mixed with 1500 large (12 mm diameter, red) spheres, and we have rotated the blend at 30 rpm for 12 revolutions. For comparison, also in Fig. 6(b), we display a snapshot of an experiment in which we rotated an equal mass blend of 1.6 mm (blue) and 4 mm (red) diameter glass beads at 16 rpm in a 1 quart double-cone blender. The banded segregation pattern shown begins to be formed in the first few rotations of the tumbler; in the experiment we have the luxury of continuing to rotate indefinitely and we find that the pattern changes little after the first several revolutions. In both experiment and simulation, the tumbler is initially filled to 50% of its total volume. In Fig's 6(c)-(d), we also show the segregation patterns in the interior of the simulation and experiment respectively. These figures display a view from the side of the blender intersecting a vertical plane through the blender's rotation axis (indicated by green arrows in Fig's 6(a)-(b)). Simulation and experiment show that the small grains form a contiguous core parallel to the axis of rotation.

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\(^5\) For simulation, see http://sol.rutgers.edu/~shinbrot/Moakher/DCseg.mpg.
Figure 6 - (a) Top view of segregation in double-cone tumbler of 12 mm (red) and 6 mm (blue) spheres after 12 revolutions at 30 rpm. (b) Top view of experiment for comparison, using 4 mm (red) and 1.6 mm (blue) glass spheres. (c) Slice through vertical plane passing through axis of rotation of blender, showing interior structure in simulation. (d) Slice through corresponding vertical plane in experiment.

To produce the experimental view in Fig. 6(d), we inserted a thin metal plate along the vertical plane of interest to divide the blender into equal halves. We then poured melted paraffin wax (indicated in Fig. 6(d)) onto the top of one half of the blender and vacuumed the grains away from the other half. Once the paraffin set, it held the grains in place so that the tumbler could be tipped with the metal plate facing up. The plate was then removed and the topmost exposed layer vacuumed to reveal the interior structure shown.

Quantitatively, we evaluate the segregation rate in the bidisperse system by plotting the intensity of segregation as a function of the number of revolutions (Fig. 7). The intensity of segregation, \( I \), is defined as:

\[
I = \sqrt{\frac{\sum_{i=1}^{n} (C_i - \overline{C})^2}{n-1}},
\]

where \( C_i \) is the concentration of one species of particle, \( \overline{C} \) is its mean concentration, \( n \) is the number of samples taken, and \( \sigma_r \) is the standard deviation expected from the same number of samples taken from a random mixture. To evaluate \( I \), we subdivide the occupied volume into cubic elements and eliminate elements containing uncharacteristically few particles; in this way we prevent spurious results caused by sampling anomalously few elements near boundaries. The fit shown in gray is to the exponential function,
\[ I = I_o - A \exp(-kN) \], in keeping with suggestions from the literature that segregation may be described by first order kinetics [Zik 1994; Cantelaube 1997; Choo 1997; Aranson 1998].

Figure 7 - Growth of intensity of segregation over time in simulation of double-cone. Gray curve is best fit to exponential function defined in text.

5. Case 2: Mixing and segregation in the V-blender

Mixing Mechanism:

The mixing mechanism that we have seen for the double-cone is straightforward: there is a co-existence between comparatively rapid convective mixing and much slower dispersive mixing. Furthermore, the velocity field is symmetric and (though significantly curved) apparently well behaved and amenable to straightforward analytic treatment.

In the V-blender this picture changes in several respects\(^6\). We begin our treatment of this problem by examining mixing of initially vertical colored stripes of identical particles. In Fig. 8 (top), we display the evolution of these stripes over the first revolution of the tumbler; beneath these plots we display the evolution of initially horizontal stripes. We stress two observations from these plots. First, particles in the yellow and cyan vertical stripes, beginning at the outward tips of the blender, end up spread along its front surfaces after one revolution; likewise the green particles beginning at the top horizontal stripe of the tumbler are brought to its front and center. This is representative of the perhaps obvious fact that the geometry of the blender enforces convective motion that contains a large axial, as well as radial, component (discussed in greater depth shortly). Second, the overall motion of grains over one cycle of rotation of this blender consists of two different operations. Particles in either shell of the tumbler are split apart at \(N \approx 0.5\), and are merged together at \(N \approx 1\). This fact, while perhaps intuitively obvious, has subtle, yet significant, consequences for mixing.

\(^6\) For simulation, see http://sol.rutgers.edu/~shinbrot/Moakher/VB.mpg.
Let us examine front and back views of identical beads colored red to the left of the plane separating the two shells of the blender and blue to the right. After two blender revolutions, front and back views are shown in Fig. 9 for the V-blender simulation (a), for a companion experiment (b) and for the double-cone simulation (c). Evidently, in the V-blender, both simulation and experiment exhibit substantially more mixing across the symmetry plane in the back view than in the front. This front/back asymmetry is not observed in the double-cone. To our knowledge, this asymmetry has not been reported previously, yet its cause appears to be central to the operation of the V-blender.

The result of the merging operation is seen in the front view of the blender, while the result of the splitting phase is seen in the back view. Thus the splitting and merging of the two halves of this blender commented upon earlier have the effect of forcing material near the symmetry plane to choose, presumably nearly at random, which blender shell to enter during every other half-period of rotation, when the shells are facing down (N = 0.5 in Fig. 8). It is during this phase of motion of the blender that mixing between the halves occurs. This holds lessons -- to be discussed elsewhere -- for blender design improvements.

Beyond this potentially applicable result, we have also detected possible signatures of intriguing dynamical structure in the V-blender's mixing flow. Although the significance of this structure remains to be determined at this stage, recent results indicating the presence of deterministic structure in other granular mixing problems [Shinbrot 1998], suggest that this observation may nonetheless be worth reporting. This structure is revealed in Fig. 10, which shows a side-view of the same horizontally layered initial state discussed previously (Fig. 8, bottom). In the side-view of this simulation, we see two structures of potential importance. At N = 0.5, in the center of the blend, we see a spiral shape, which is likely just a reflection of the overturning of the granular mass about a central core. Nevertheless, the presence of this spiral indicates that there must be significant shear in this region. Since shear tends to expel larger particles [Lacey 1943, Leighton 1987], we expect to find high concentrations of fines here, and indeed this is what we observe in experiments using polydisperse particles. Second, at N = 1, we observe a diagonal swath of connected blue particles, which originated at the bottom of the tumbler. Since the swath remains contiguous, despite being evidently stretched along a particular direction, this reinforces the suggestion made elsewhere [Shinbrot, 1998] that properties of chaotic mixing of fluids -- such as asymptotic directionality [Alvarez, 1998] and the view of mixing as being representable by repeated iterations of simple topological operations [Aref, 1984] -- may
carry over into the study of mixing of grains. Clearly more work is needed to substantiate this physical picture, but our simulations seem consistent with this view.

Figure 9 - Top: front views of V-blender and double-cone; bottom: back views. (a) V-blender simulation after 6 revolutions, with identical particles initially blue to the right of the symmetry plane and red to the left. (b) Experiment in transparent V-blender, again after 6 revolutions. In both simulation and experiment, more mixing is apparent in view from back than from front. (c) By comparison, simulation of double-cone reveals nearly symmetric mixing from front and back after 6 revolutions.

Figure 10 - Evolution of identical spheres initially layered horizontally. These views show a slice through the center of one shell of the tumbler.

Velocity Fields:

As with the double-cone blender, it is instructive to examine the velocity field within the V-blender. This is shown in Fig. 11, where each snapshot represents velocities of particles whose centers lie within a square lattice of cells averaged over the same phase during the final 4 cycles of tumbler motion. On the left, we see the velocity field in planar slices traveling through the center of one shell of the blender. Importantly, the velocity field here does not consist of a flowing layer above a solid-body region, as was seen in the double-cone and as one might surmise from existing literature. Instead, transport consists of
phases of motion during which nearly all of the grains are in motion, either splitting from
the blender center toward its arms (upper right of Fig. 11) or merging from arms to center
(lower right). This is in keeping with the picture described earlier in this section, but the
velocity fields in addition reveal that between these two stages, the granular bed is largely
static. Thus flow in the V-blender consists of intermittent periods of near-universal motion
of particles alternating with periods of near-universal stationarity. This agrees with
experimental observations, in which we have used transparent vessels, and where similarly
we see rapid sloshing motions of particles alternating with periods of substantial calm.

We quantify this behavior in the inset to Fig. 15 for both double-cone and V-blender. Here
we plot the fraction of particles traveling with speed \( V > V_c \) as a function of the phase of
rotation of the tumbler, \( \tau \). For this plot, we choose a value for the cut-off speed, \( V_c \), of 3
times the time-averaged speed of all particles. From this inset plot, we confirm that
throughout the rotation of the double-cone, a small (8\%) and nearly constant fraction of
particles flow 'rapidly' by this measure, while for the V-blender, there are strong
fluctuations between a state in which < 5\% of particles flow rapidly and a state in which >
25\% of particles flow rapidly. If \( V_c \) is taken to be the mean particle speed, the sloshing
motion in the V-blender causes up to 68\% of particles to periodically move with speed >
\( V_c \).

By examining the snapshots to the right of Fig. 11, taken at opposite phases of the tumbler
rotation during which collective particle speeds are near their maximum, we see directly the
alternating splitting and merging of the granular bed that appears to govern mixing in this
flow.

\[ \text{Figure 11 - Left plots: side view of velocity field at successive phases,} \]
\[ \tau = N \mod 1, \text{ of rotation of V-blender. These views show a slice} \]
\[ \text{through the center of one shell of the tumbler; each plot displays the} \]
\[ \text{velocity field averaged over space and time as described in text. Right} \]
\[ \text{plots: top views of velocity field taken on horizontal slices through} \]
\[ \text{blender at} \tau = 1/4 \text{ and} \tau = 3/4, \text{ when mean speeds are maximal. The} \]
\[ \text{arrows in the right plots indicate the plane through which velocities in} \]
\[ \text{the left plots are taken.} \]
Quantification of Mixing Rate:

Different as this velocity field is from simpler blenders such as the drum or double-cone, it comes as little surprise that the mixing behavior of the V-blender is unique. In Fig. 12, we plot the change in variance with time in a monodisperse simulation of 20,000 3.2 mm diameter particles tumbled at 30 rpm. Unlike the double-cone (Fig. 4), this tumbler seems to lack a simple exponential scaling region, and the merging/splitting action is manifested by strong oscillations in the variance. On the other hand, neglecting the barrier to transport across the symmetry plane commented on earlier, the mean rate of mixing is much faster in the V-blender than in the double-cone: the same decrease in variance seen for the double-cone in 6 revolutions is achieved in the V-blender in under 1 revolution. Likewise, by the third revolution the V-blender has nearly achieved its asymptotic mixed state. Note that the light gray curve in the main plot has a different asymptote than the other curves because there are fewer light-colored particles than represented by the other shades.

Figure 12 - Main plot: logarithm of variance of volume elements in V-blender. Three sets of initial conditions -- outermost (light gray), central (gray) and innermost (black) -- are displayed in these calculations, as identified in the lower inset. Upper insets show concentrations of particles originating in outermost, central and innermost regions after 0, 3, and 6 revolutions.

Segregation in the V-blender:

We can see the effect of splitting and merging on dissimilar particles in Fig. 13(a). Here we display the evolution of an initially randomly placed arrangement of 16000 small (2.5 mm diameter, blue) and 2000 large (5 mm diameter, red) spheres. After 6 revolutions at 15 rpm, an asymmetric segregated state is established. From the front, in this state the smaller particles appear to have migrated outward. In a view from the back, however, it becomes clear that something more complicated is going on: the smaller particles are seen to accumulate near the center. This is seen in the enlarged view in Fig. 13(b). A slice

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7 For simulation, see http://sol.rutgers.edu/~shinbrot/Moakher/VBseg.mpg.
through the center plane of the tumbler (Fig. 13(c)) shows that the small particles actually occupy a horizontally oriented, V-shaped, region.

Comparisons between simulation and experiment in both upright and inverted orientations of the tumbler are shown in Fig. 14. For the experiments, we half filled a transparent 1 quart V-blender with equal masses of 1.6 mm (blue) and 4 mm (red) glass spheres and rotated the blend at 16 rpm until an asymptotic segregation pattern was formed. As with the double-cone experiment, the segregation pattern forms rapidly and changes little after several revolutions of the tumbler. Evidently, in the upright configuration, Fig's 14(a)-(b), the horizontal V-shaped segregation pattern is seen in both experiment and simulation. As anticipated earlier (cf. Fig. 10), in both experiment and simulation, a region of finer particles emerges close to the axis of rotation. There is likewise correspondence between experiment and simulation for the inverted orientation of the tumbler (Fig's 14(c)-(d). In this orientation, however, the smaller particles arrange themselves into two symmetric wedges at the extremes of either shell.
We have also evaluated the growth in intensity of segregation, I, for the V-blender, as displayed in Fig. 15. As before, a fit to the exponential function $I = I_0 - A \exp(-k N)$ is shown in gray. For the V-blender, there appears to be a variation in I with period roughly one per revolution of the tumbler. In Fig. 15 we indicate approximate times at which merging of particles from opposite tumbler shells occurs. These times roughly correspond to minima in I (which are absent from the double-cone plot: Fig. 7). This suggests that while mixing of similar size particles across the symmetry plane occurs predominantly during splitting of particles into the blender shells, mixing of dissimilar size particles occurs at the opposite phase of motion, during merging from the shells. This is intriguing and indicates the need for more detailed dynamical studies, specifically focused on splitting and merging as occurs both in tumblers such as the V-blender and in chutes and near baffles.

![Figure 15 - Main plot: growth of intensity of segregation over time in simulation of V-blender. Gray curve is best fit to exponential function defined in text. Arrows indicate approximate times at which merging of particles from tumbler shells occurs. Inset: fraction of rapidly moving particles vs. phase of tumbler rotation for double cone and V-blender.](image)

6. Conclusion:

We have presented results from the first fully three-dimensional simulations of two industrially relevant tumblers: the double-cone and the V-blender. It appears from these studies that the detailed dynamics in these blenders are quite different, yet both share the same mixing bottleneck: dispersion across a plane of symmetry. Segregation in these tumblers likewise share commonalities, with asymptotic segregated patterns becoming established quite rapidly and persisting indefinitely. The patterns themselves look rather different, yet effectively serve the function of separating large and small particles into zones with substantially different velocity fields, each compatible with a particular subspecies.
Several avenues are indicated for future research. First, a lesson from mixing of fluids is that wherever possible symmetries should be broken. This lesson carries over directly to granular research, and implies that new tumbler designs incorporating broken symmetries -- for example through judicious baffle design [Brone 1997], temporal perturbations [Wightman 1997; Brone 1997, 1998a], or reshaping the geometry of the tumbler [Chang, 1992] -- are merited. Second, tumblers such as the V-blender appear to function very differently from tumblers like the double-cone. In particular, the V-blender operates intermittently, combining splitting and merging, while the double-cone operates nearly continuously, with a nearly constant flow of particles in a more uniform surface layer. This results in significantly more rapid mixing in the V-blender than in the double-cone; nevertheless both tumblers exhibit reproducible and rapidly occurring segregation patterns. Initial inroads have been made into modeling of constant [Khakhar 1997] or near constant [Shinbrot 1998] tumbler flows, but strongly intermittent granular flow is much less well understood. Since a large class of practical tumblers appear from this work to operate intermittently, research in this field is also warranted. Likewise, this intermittency indicates that hybrid and continuum approaches, while justifiable for some blenders, must be viewed with caution for others. Finally, from a fundamental standpoint, the apparent facts that similar particles mix during mechanical separation and dissimilar ones mix during recombination are unexpected and deserve more detailed study.

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References:


Aranson, I. & Tsimring, L.S. 1998 "Dynamics of axial separation in long rotating drums" (preprint, Argonne National Laboratories)


Mehta, A. (Editor) 1994 Granular Matter, an interdisciplinary approach, (Springer-Verlag, NY)


