RUTGERS - THE STATE UNIVERSITY OF NEW JERSEY
DEPARTMENT OF CHEMICAL & BIOCHEMICAL ENGINEERING

155:307 Chemical Engineering Analysis II — Final Examination
Dr. A. Constantinides — Fall 2000

PLEASE WRITE YOUR NAME HERE:____.
AT THE END OF THE EXAMINATION, PLEASE RETURN THE EXAM SHEET TOGETHER WITH THE BLUEBOOK. BLUEBOOKS NOT CONTAINING THE EXAMINATION SHEET WILL BE PENALIZED 10 POINTS.

Problem 1. (8 points)

A is an $(n \times n)$ matrix, **x** is an $(n \times 1)$ vector, **c** is an $(n \times 1)$ vector, and **0** is an $(n \times 1)$ vector of zeroes. Under what condition(s) does the following set have a unique solution?

a) A homogeneous set of linear algebraic equations: **Ax = 0**
b) A nonhomogeneous set of linear algebraic equations: **Ax = c**

Under what condition(s) does the following set have an infinite number of solutions?

c) A homogeneous set of linear algebraic equations: **Ax = 0**
d) A nonhomogeneous set of linear algebraic equations: **Ax = c**

Problem 2. (12 points)

Integrate the function $$f(x) = e^{-0.1x}$$

in the range $0 \le x \le 6$, using a stepsize h = 1, with the following methods (calculate all values to four decimal places):

(a) Trapezoidal Rule
(b) Simpson's 1/3 Rule
(c) Compare these results with those obtained from the analytical solution. Explain the differences, if any.

Problem 3. (25 points)

For the following chemical reactions

$$\begin{array}{lcl} A & \xrightarrow{k_1} & B \\ \downarrow k_2 & & \\ C & & \end{array}$$

the rate of change of the concentration of component *A* is described by the differential equation

$$\frac{dC_A}{dt} = -k_1 C_A - k_2 C_A^2$$

where

C_A = concentration of A (moles/liter)

k_1 = rate constant = 2.0 hr^{-1}

k_2 = rate constant = 1.0 hr^{-1}(moles/liter)$^{-1}$

The initial concentration of *A* is:

C_A (0) = 1.0 mole/liter

Determine the concentration of *A* at *t* = 1/4 hr, 1/2 hr, 3/4 hr, and 1 hr, using the Runge-Kutta 2nd-Order method, with *h* = 1/4 hr.

Problem 4. (30 points)

The steady state temperature of a thin square plate with internal heat generation is given by the following partial differential equation:

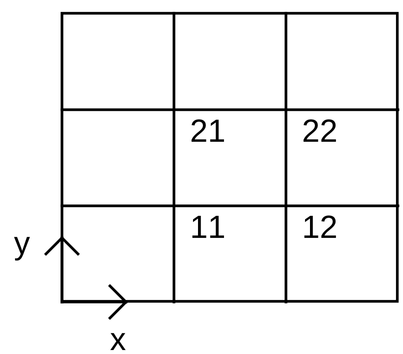
$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+R=0$$

where: $\boldsymbol{R}$ = heat generation = 90°F/hr.
$\boldsymbol{\alpha}$ = thermal diffusivity = 0.5 ft^2/hr.

The size of the plate is 1 foot × 1 foot. The boundary conditions are:

T(0, ***y***) = 1500°F ***T***(1, ***y***) = 100°F
T(***x***, 0) = 600°F ***T***(***x***, 1) = 40°F

Divide the plate into nine squares of dimension 1/3 foot × 1/3 foot, and label the four internal points as shown below:



(a) Using finite differences, set up the above differential equation for numerical solution. Show all equations needed to evaluate the temperature at the four internal points.

(b) List three methods that may be used for the solution of the finite difference equations to evaluate the temperature at the four internal points.

(c) Apply the Gauss-Seidel method to evaluate the temperature at the four internal points. Explain all steps. Do only three (3) iterations and round the numbers to the closest degree. **Do not use the overrelaxation method.**

(d) Estimate (as accurately as possible) the steady state temperature at the exact center of the plate.

Problem 5. (25 points)

The following chemical reaction takes place in a batch reactor:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

At t = 0, the concentrations of the three components are:

$$C_A(0) = 1.0, \quad C_B(0) = 0.5, \quad C_c(0) = 0.3$$

a) (10 points) Assuming that the mechanism of these reactions follows monomolecular kinetics, write the equations which describe the rate of change of the concentration of the three components.

b) (15 points) Find the analytical solution of these equations using the eigenvalue-eigenvector method. The values of the constants are:

$$k_1 = 1.0 \text{ and } k_2 = 2.0$$