

Granular Flow Transitions on Sinusoidal Surfaces

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Abstract

We report on a newly discovered bifurcation that occurs in dry grains flowing down a chute with a wavy bottom. We find that the bifurcation outwardly resembles the long-known fluid analog of inviscid channel flow over a wavy bottom reported in 1886 by Lord Kelvin, however in detail the two situations differ significantly. We compare three distinct states seen in the granular system: a ‘regular’ flow in phase with the bottom wave, an ‘antiregular’ flow that is out of phase, and a ‘flat’ flow in which the surface slides nearly uniformly downhill. Additionally, we discuss evidence that sustained subsurface circulation in the granular bed, accompanied by strong fluctuations in flow velocities, can appear in granular flows over wavy surfaces.

Background

Over a century ago, Lord Kelvin predicted a surface wave bifurcation for fluid flow through a channel with a wavy bottom (Thomson, 1886; Lamb, 1932). This bifurcation defines a transition between ‘regular’ flow, in which the surface is in phase with the wavy bottom, and ‘antiregular’ flow, in which it is out of phase. Since that time, experimental and computational studies have investigated the behavior of fluid flow over wavy surfaces (Krettenauer and Schumann, 1992; Gong et al., 1996; Hudson et al., 1996; Nakayama and Sakio, 2002; Günther and Rohr, 2003), shear in this geometry (Benjamin, 1958), and mechanisms of wave generation in two-phase wavy flows (Montalbano and McCready). Although the observations made by Lord Kelvin for fluids have stood for a century, no studies to our knowledge have investigated similar transitions in granular flow on wavy bottoms. Yet the precise problem of granular flow over wavy and rough bottoms is encountered in both laboratory (Takahashi, 1981; Savage and Hutter, 1989; Iverson, 1997; Pouliquen, 1999; Hanes and Walton, 2000; Denlinger and Iverson, 2001; Louge, 2003; Louge, 2003) and geophysical debris (Hakonardottir et al.; Iverson, 1997) flows, and there are theoretical suggestions from simulations of single balls bouncing down a bumpy incline (Dippel et al., 1996) that such transitions may occur as well. In this article, we perform, to the best of our knowledge, the first experimental investigations of this problem, and we find that there is indeed a sharp transition between regular and antiregular granular flow as the angle of the wavy chute is varied. The granular transition

differs in important ways from the fluid one, and we propose a mechanism involving circulatory subsurface flows that seems to capture some of the essential kinetics involved.

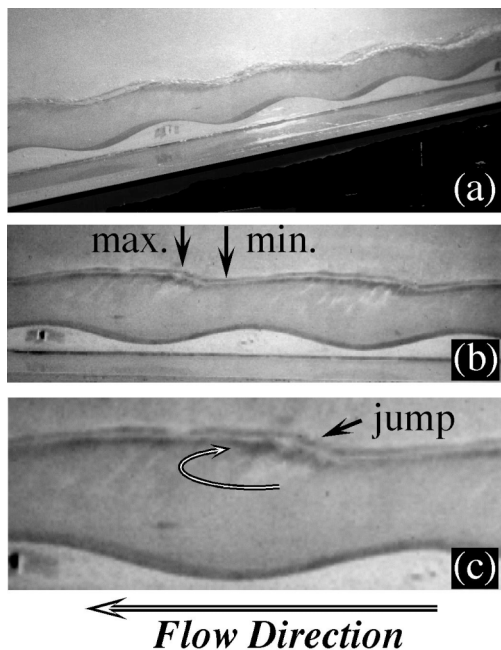
Studies of shear in granular materials date at least back to Bagnold's pioneering research of desert sand transport over half a century ago (Bagnold, 1936), with related geophysical reports appearing more than a century before that (Bishop, 1713; Lyell, 1858; Hopkins, 1862). More recently, research into shear in well-instrumented cells (Rabinowicz, 1956; Miller et al., 1996; Menon and Durian, 1997; Howell and Behringer, 1999; Losert et al., 2000; Mueh et al., 2000), chutes (Savage and Hutter, 1989; Hanes and Walton, 2000; Forterre and Pouliquen, 2001; Goldfarb et al., 2002; Gray et al., 2003; Louge, 2003) and other systems (Hsiau and Hunt, 1993; Komatsu et al., 2001) has attempted to clarify the mechanisms of granular response to controlled stresses. Notwithstanding these investigations, granular beds often display solid-like and fluid-like qualities side-by-side within a single experiment, and generally applicable rules underlying granular phenomena remain elusive (several comprehensive surveys are available from a variety of perspectives, e.g. (Bridgwater, 1976; Savage et al., 1983; Jenkins, 1986; Johnson et al., 1990; Jaeger et al., 1996; Iverson, 1997; Goddard and Alam, 1999; Kadanoff, 1999)). We know that these behaviors have important practical ramifications. On the laboratory scale, periodic disturbances of granular tumbling flows have been shown to result in improvements in mixing rates by orders of magnitude (Shinbrot et al., 1999), and in a provocative example on geophysical scales, a periodic array of stone barriers have recently been erected to dissipate the destructive force of snow avalanches in the town of *Neskaupstadur* in eastern Iceland (Hakonardottir et al.; Johannesson and Hakonardottir). Moreover, recent hydrodynamic treatments (Gray et al., 2003) have shown great promise in chute flows. In the present paper, we seek to advance the understanding of granular transitions in flows across wavy bottoms: a topic whose understanding lags considerably behind that in the seemingly closely related field of fluid dynamics (Hakonardottir et al.; Johannesson 2003; Silbert 2002; Hewgill et al., 1981; Bocquet et al., 2002).

Phenomenology

As we have emphasized, fluid flows over wavy surfaces have been studied for many years, both as paradigmatic theoretical problems and because practical systems are seldom uniform (Iverson, 1997). One such system appears in channel flow across a wavy surface, as shown in Fig. 1 from a fluids experiment in our laboratory. In fluids experiments, channel flow undergoes a hysteretic bifurcation from the 'regular' state at higher flow rates (Fig. 1(a)) where the fluid free surface is in phase with the (sinusoidal) channel bottom to the 'antiregular' state at lower flow rates (Fig. 1(b)), where the free surface is out of phase with the channel bottom. This bifurcation coincides with two dynamical features of interest: the first is the emergence of a hydraulic jump (Stoker, 1957) (indicated in Fig. 1(c)), the second is the creation of a recirculation loop (also indicated) just downstream of the jump. This bifurcation is very robust, and

persists despite being removed from the ideal, potential-flow, limit of vanishing viscosity (our Reynolds number is ~ 100).

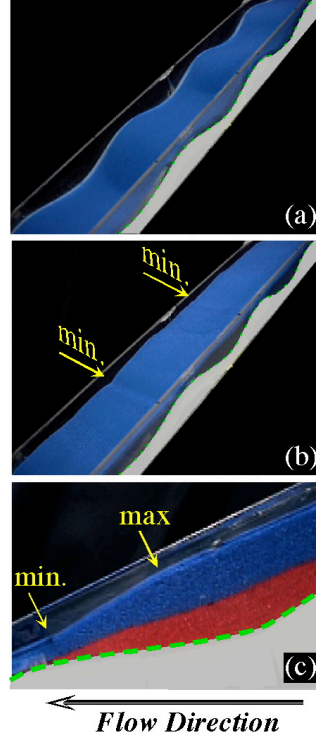
Figure 1 - Snapshots of steady (a) 'regular' flow and (b) 'antiregular' flow of water in a horizontal channel with a sinusoidal bottom. As indicated in (b), in the antiregular state the maximum of the free surface is over the minimum of the channel bottom. (c) Enlarged view of antiregular flow showing hydraulic jump. The channel is 5 cm wide, 244 cm long and is fed from the right; the wavelength and peak-to-peak amplitude of the sinusoidal insert are 30 cm and 2.5 cm respectively.



To investigate a granular analog of this bifurcation, we construct a 244 cm long, 10.2 cm wide chute in which we fit a variety of amplitude and wavelength sinusoidal bottoms. Chutes notoriously tribo-charge as dissimilar materials rub across the chute surface (Shinbrot, 1985), so the bottoms (which are cut into sinusoidal shapes from wood slats) are covered with smooth, grounded, aluminum flashing to provide a reproducible and minimally charged surface on which to perform experiments. The sides of the chute are clear acrylic to permit us to see near-wall flows, and to further limit charging, we apply anti-static fluid to these sides before each experimental trial. In Fig. 2, we display characteristic results, here for a sinusoidal bottom with wavelength 20 cm and peak-to-peak amplitude 2 cm. Art sand (colored irregular glass grains of mean diameter 0.5 mm) is released onto the chute from an upstream hopper whose outlet is 10 cm above the chute surface, and whose mass flow rate is measured over a period of minutes to remain stable at 281 ± 2 g/s.

From this figure, we see that there indeed is a transition from regular (Fig. 2(a)) to antiregular (Fig. 2(b)) flow, here as the angle of incline of the chute is reduced from 32.7° to 31.2° as measured with a digital level. As with the fluid case, there appears to be an identifiable minimum (indicated in Fig. 2(b)) in the antiregular state, possibly accompanied by an upstream hydraulic jump. The minimum and nearby maximum are identified in the enlarged side view of Fig. 2(c). In the latter figure, we also use two differently colored, but otherwise identical, grains to study transport: this is discussed in a later section.

Figure 2 - Snapshots of (a) 'regular' flow (at 32.7° chute inclination) and (b) 'antiregular' flow (at 31.2° inclination) of dry sand in a tilted channel with a sinusoidal bottom. As indicated in (b)-(c), the peak of the free surface is over the trough of the channel bottom in the antiregular state. (c) Enlarged view of antiregular flow showing that a portion of the bed that was released earlier (red) is eroded more slowly than flowing grains released subsequently (blue), which travel mostly near the surface. There appears to be a hydraulic jump between the 'min' and the 'max' arrows in (c) (see text). The channel is fed from a hopper to the right of the photographs. The sinusoidal bottom is identified by the dashed green line.



Transients and Final States

We performed several independent experiments to evaluate the dependencies of the granular form of this surface wave bifurcation. First, we measured the length of time, τ , taken to achieve a steady final state as a function of chute inclination angle, θ . To make our measurements reproducible, we opened the hopper onto a clean chute at a fixed inclination angle, and evaluated the time taken until the central 6 wavelengths (excluding one wavelength at the top and one at the bottom where there are some variations due to end effects) of the chute all exhibited the same state.

Flow states in the wavy chute are significantly hysteretic, however the sequence of events that typically occurs after opening the hopper is as follows. When the hopper is first opened, grains relax to fill the valleys of the chute. A snapshot showing this process while it is occurring is shown in the lower inset to Fig. 3(a). In this snapshot, grains flowing from uphill (to the right) cascade in a jet (cf. Hakonardottir, 2003) over the top of an upstream sinusoidal peak to impinge on the downstream end of the next peak (black arrowheads). The formation of a jet occurs when the inlet kinetic energy of the flowing grains is very high (as occurs when the inlet hopper is raised); if the kinetic energy is lower, the grains initially cover the entire chute in a uniform blanket that gradually thickens in the troughs until the final state is reached. In the case where a jet appears, grains in the jet fly nearly ballistically over the underlying bed. When these grains collide with the downstream surface, they lose kinetic energy and subsequently collapse into the

nearest valley. During this collapse, the front of the jet (indicated by arrowheads in the inset) moves downhill until the upstream valley is filled with debris from prior collisions with the downstream surface.

At this point (which is reached either with an initial jet, or for less energetic inlet speeds without the jet) one of three steady states emerge, depending on the chute inclination. If the chute inclination is greater than about $\theta_{c1} = 32.3^\circ$, the static grains in the valley are washed downstream by the continuing influx of grains flowing downhill, to leave a regular surface. If the inclination is below this value, a more complex flow is produced, in which the surface flows downhill but, as we demonstrate shortly, subsurface flows recirculate (Gray, 2003) to produce the surface minima shown in Fig. 2(b). Finally, if the chute inclination is below about $\theta_{c2} = 31^\circ$, the valleys fill completely to produce a flat static bed at the height of the sinusoidal peaks, and inflowing grains flow slide with nearly uniform depth and speed on this bed.

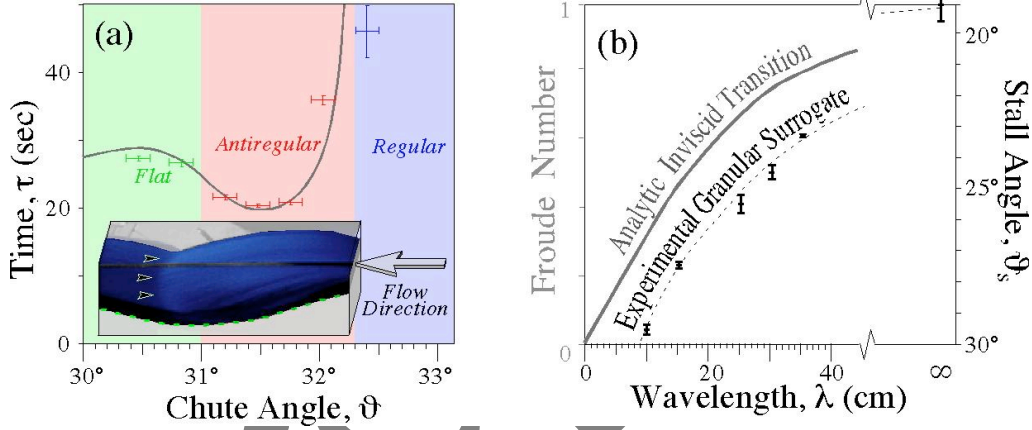


Figure 3. (a) Time to reach steady state vs. chute angle. The data points are averaged over 10 separate trials. Angles greater than about 32.5° attain the regular state almost immediately. Colored regions indicate flat, antiregular, and regular regimes described in text; boundaries between these regimes are precise in our experiment, but change with grain properties and environmental conditions and should therefore be treated as approximate. The inset shows a snapshot of the evolving flow in an antiregular state (see text). (b) Plots of fluid (solid line) and granular (dashed) transitions between regular and antiregular flow vs. wavelength of sinusoidal bottom. For the fluid case, we plot $Fr = \tanh(kd)/kd$, where d is the depth of the flow, taken to be 5 cm, and k is the wave number, $2\pi/\lambda$. For the granular case, higher kinetic energy flows stall at lower chute angle (see text), so for comparison we plot decreasing stall angle (right ordinate) alongside increasing Froude number (left). Note that because the scales are different, any agreement is at most qualitative.

Corresponding to each of these states is a time, τ , required to reach the final state, as we defined previously. In the main part of Fig. 3(a), we plot measurements of τ at chute angles in the range 30° - 33° . Each measurement is an average over 10 separate experimental trials. Close to the transition angle of about $\theta_{c1} = 32.3^\circ$ the surface invariably starts out in a regular, in-phase, state. Slightly below θ_{c1} , the most

downstream valley fills with grains, after which the surface in that valley slowly transitions to an antiregular state. This appears to cause grains in the next valley upstream to slow: that valley then becomes antiregular as well, and so forth up the chute. The regular-antiregular transition appears to display significant critical slowing down (Howell and Behringer, 1999): as the chute angle gets closer to ϑ_{c1} , the flow takes increasingly longer to settle into the final state reported. By contrast, the transition between antiregular and flat surface flows around $\vartheta_{c2} = 31^\circ$ seems to be continuous.

To aid the eye, we have correspondingly plotted in gray the function indicated in the figure containing a tanh to fit the transition at ϑ_{c1} , and a square-root singularity to fit the apparent slowing down (explicitly, the curve shown is: $\tau = [2 - \tanh(2(\vartheta - \vartheta_{c1}))]/2\sqrt{|\vartheta - \vartheta_{c2}|}$). We stress that this is not a theoretical model: the constants are chosen strictly for convenience, and we present this function only as a curve fit that seems to roughly define the experimental phenomenology that we observe – i.e. one smooth continuous transition at ϑ_{c1} followed by a second showing slowing-down at around ϑ_{c2} . We compare these phenomenological descriptions with other data shortly.

Parametric Dependence

In a second set of experiments, we varied the wavelength of the sinusoidal chute bottom. We additionally investigated doubling the amplitude of the sinusoidal chute shape, however this had the effect of stalling all motion in the chute valleys, which in turn eliminated the antiregular state. We therefore kept the amplitude at the lower value of 2 cm peak-to-peak and varied the sinusoidal wavelength. Results from these experiments are shown in Fig. 3(b). Here we fabricated several sinusoidal inserts with wavelengths ranging from 10 to 40 cm. For a given wavelength insert, we maintained a constant inflow rate (281 ± 2 g/s as before), and beginning at a steep angle, we slowly decreased the chute angle until the flow stalled. For each of the wavelengths shown, the surface went through a regular, antiregular, and flat state as indicated in Fig. 3(a). These transitions are somewhat variable, as indicated by the error bars in Fig. 3(a). By comparison, the stall angle, ϑ_s , seemed to be more reproducible and represents a fixed upper bound on the other transitions, so we measured ϑ_s , which is shown in Fig. 3(b) as a surrogate for the other transitions. Each data point here is the average of 5 experimental trials at the specified wavelength. The dashed curve through the data is an exponential fit that asymptotes to the flat chute value of about 19° (appropriate to our chute and grains) as the wavelength $\rightarrow \infty$.

Also in this plot, we include a curve (solid line) of the Froude number as a function of wavelength predicted from Kelvin’s analysis of an inviscid and irrotational fluid. For fluid flow over a wavy bottom, the Froude number is defined in the usual way as a ratio of kinetic to potential energies:

$$Fr_f = \frac{v^2}{2gd}, \quad [1]$$

where v is the mean surface velocity of the fluid, g is gravity, and d is the characteristic depth of the flow. Kelvin (Thomson, 1996; Lamb, 1932) described a transition between regular and anti-regular flow at the critical Fr :

$$Fr_f^* = \frac{\tanh(kd)}{kd}, \quad [2]$$

for a sinusoidal bottom of wavenumber k . To provide a comparable hydrodynamic treatment of our chute flow, we need to define relevant kinetic and potential energies (Gray, 2003). The kinetic energy per unit mass of grains can be established directly from measurements of the granular inflow, as described below. The relevant potential energy is not entirely unambiguous, however, since any of several length scales could be used to calculate it. Prior work on granular flow down inclines has shown that the maximum slope controls the speed of the flow (Louge, 2003; Savage, 1989; Hanes, 2000); this slope is closely approximated in our system by $h \cdot \sin(\vartheta)/\lambda$, where h is the depth of the sinusoidal bottom, λ is its wavelength, and ϑ is the angle of inclination of the chute. The vertical length scale that is most germane to calculating the potential energy is thus approximately $h \cdot \sin(\vartheta)$, and so we define the Froude number for the wavy granular flow to be:

$$Fr_s = \frac{(Q/\rho A)^2}{2gh \sin \vartheta}. \quad [3]$$

Here Q and A are respectively the measured mass flow rate and orifice area at the inlet to the chute, ρ is the density (which we hold to be the bulk density of our glass beads, 2.5 g/cc), and g is gravity.

The chute angle, ϑ , appears in the denominator, and so Fr increases as ϑ decreases. Correspondingly, in Fig. 3(b), we plot Kelvin's analytic curve for fluids for *increasing* critical Froude number (left ordinate), alongside our data obtained for grains for *decreasing* stall angles (right ordinate). As we have stressed, the stall angle is a convenient and reproducible surrogate for the other transitions seen, and does not in closed form predict the critical Froude number. Nevertheless, Fig. 3(b) indicates that despite obvious differences between the simplified inviscid and irrotational fluid model and the strongly dissipative (Goldhirsch and Zanetti, 1993; Du et al., 1995; Henrique et al., 1998) (and as we will show, significantly circulatory) experimental granular flow, transitions in both cases tend toward higher kinetic energy at longer wavelength.

Phase Diagrams:

To more fully explore the parametric dependencies of the flow transitions in the wavy chute, we also varied the incoming flow rate to the chute. For this purpose, we used a wavy chute of fixed wavelength, 20 cm, and peak-to-peak amplitude, 2 cm, and we investigated 108 parameter sets consisting of 12 chute inclinations and 9 flow rates. Flow rates were set by inserting plates with fixed diameter orifices at the end of the inlet hopper; for each orifice size, we measured the flow rate as before, and in Fig. 4(a), we display how the flow state depends on inclination and flow rate.

The state of the flow was established in two ways. First, we determined by visual inspection whether the free surface was in phase with the sinusoidal bottom (for regular flow) and, if not, whether the surface had a minimum (for antiregular flow) or not (for flat flow). Second, we measured the volume of grains (the “hold-up”) contained in each cycle of the sinusoidal chute, which we found to be a signature of the state that correlates strongly with visual observations. For the flat state, the hold-up is 440 ± 20 ml; for the antiregular state the hold-up is 390 ± 20 ml; and for the regular state, the hold-up varies between 50 ml and 150 ml, depending on flow rate. The uncertainties are estimates based on the accuracy of measuring flow heights (and so the hold-up) through the transparent side-wall of the chute.

From Fig. 4(a), we see that distinct flat, antiregular and regular states appear for low flow rates, but at flow rates above about 500 g/s (point “P” in the figure), the antiregular state vanishes and the flow undergoes a transition between a flat state, at low chute angles, and a regular state, at higher angles.

By comparison, in Fig. 4(b), we plot the Froude number calculated using [3] as the chute inclination and flow rate are varied. Also in Fig. 4(b), we plot the plane $Fr_s = 1$ and we superimpose the states from Fig. 4(a). Apparently the anti-regular state disappears above point “P”, i.e. above about $Fr_s = 1$. The disappearance of the anti-regular state, which contains a hydraulic jump as shown in Fig. 2(b), is to be expected above $Fr = 1$, where hydraulic jumps cannot be sustained. The source of the λ -shaped antiregular boundary in Fig’s 4(a)-(b) is not so readily explained, however.

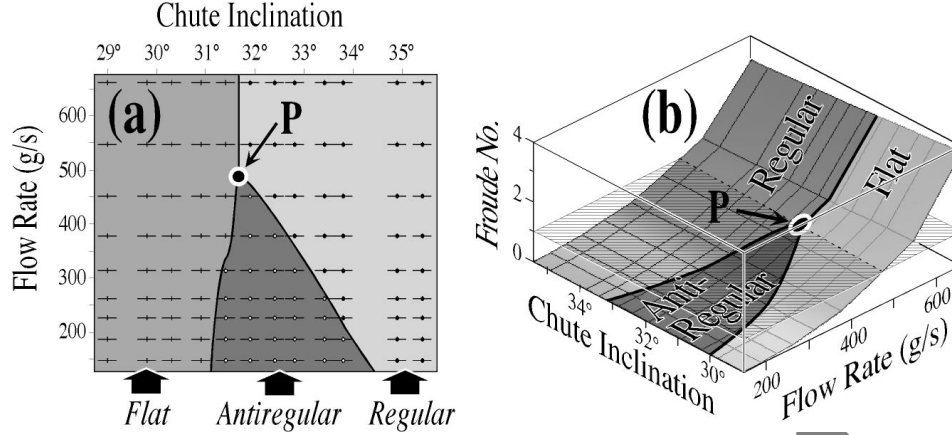


Figure 4 – Phase diagram, showing the effect of inflow rate onto the chute and chute inclination on the flow state. (a) Data are shown for 9 values of flow rate and 12 values of inclination, and error bars are as shown. The transition lines are approximate; the state changes at the nearest datum on either side of the lines. (b) For the same flow rates and inclinations, we plot the Froude number calculated using Eq. [3], and we superimpose the transition curves from (a). We also show a hatched plane at $Fr = 1$: above $Fr = 1$, hydraulic jumps cannot be sustained, and the antiregular state (which contains a hydraulic jump, cf. Fig. 2(c)) disappears.

Velocimetry

We therefore seek to unveil the mechanism underlying the granular transition in greater detail. For this purpose, we have performed Particle Image Velocimetry (PIV) (Wightman et al., 1995), evaluating surface particle velocities in the three states indicated in Fig. 3(a). These data, discussed next, confirm that the flat, antiregular, and regular states are indeed distinct, and are signatures of very different internal mechanisms of energy dissipation in the granular flow that are not well captured by averaged measures such as the Froude number. Results of these experiments are shown in Fig. 5 for PIV taken from above in each of the three flow states. In each plot we show in gray the location of the sinusoidal trough beneath. Velocity fields are averaged over 2 seconds (1000 high speed video frames) in each case.

These velocimetry data provide several notable findings. First, the regular state exhibits an increase in speed at the upstream aspect of the trough (2.8 m/s average as indicated), and a decrease at the downstream aspect (1.1 m/s). This is as one would expect: in the regular state, grains follow the contour of the underlying chute (cf. Fig. 1(a)), and accelerate in the regions of steepest gradient and slow as the gradient decreases. The flow is qualitatively different in the antiregular state: here as shown in Fig. 2(b), the flow is thin – and rapid – near the crests of the underlying chute, and is thicker – and slower – in the troughs. The surface flow throughout the trough region is both more uniform and substantially slower (between 0.1 and 0.5 m/s) than in the regular case, and flow actually accelerates (to about 0.5 m/s) above the crest of the sinusoidal insert.

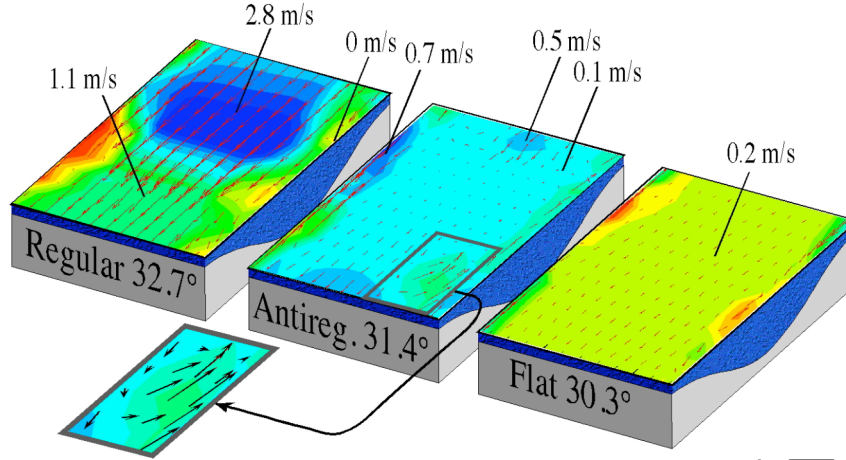


Figure 5 – Surface velocity fields, averaged over 1000 frames, during regular, antiregular, and flat flows. In the regular case, there are no surprises: flow is rapid and accelerates upstream of valleys and slows downstream. At the other extreme in the flat case, flow is an order of magnitude slower, and, since valleys are filled with stalled grains, is very uniform. In the intermediate antiregular state, surface flow speeds are locally variable, and sometimes even display circulatory eddies as in the enlarged inset. Note from the identified speeds that colorscales differ in the three cases.

Second, as indicated by sample numerical data above each plot in Fig. 5, quantitative surface speeds are reduced by about an order of magnitude in the antiregular state as compared with the regular state. Since mass flow rates are held constant, this implies that *the flowing layer must be an order of magnitude thicker* in the antiregular state than in the regular one.

Finally, the flat state is measured to be essentially independent of location, and by all appearances seems to consist of a slowly and uniformly sliding layer on a flat stationary bed that is formed when the sinusoidal troughs fill with static grains. Noting again that the mass flow rate is held fixed in all three cases, we can use the flat state to serve as a point of reference for the regular and antiregular states. That is, the regular steady flow must be much thinner than the flat state in order to conserve mass, and by the same token in the antiregular state, the surface height is nearly identical to that in the flat state, so the spatially averaged flow speed must be the same in the two states, i.e. about 0.2 m/s. Since most of the surface flow above the crest in the antiregular state is at a lower speed (about 0.1 m/s as indicated), it follows that above the crest there must be local regions that travel significantly faster.

Thus mass conservation implies that in the example shown in Fig. 5, there must be a confined high speed jet passing over the crest in the antiregular state (cf. Fig. 3(a), inset). This jet is manifested in a localized higher speed region (0.5 m/s indicated) and additionally as this jet enters the slower moving flow near the trough, it appears to produce a circulatory flow, as shown in the enlarged inset. PIV data here clearly confirm that circulation is present in the antiregular state, whereas we have never witnessed such a

circulatory flow in our experiments of the flat or regular states. We remark that the locations of the jet and the presence of circulation are themselves variable: in some PIV experiments, surface eddies are seen and in others they are not. Additionally, it seems likely that the eddies that we saw are a function of the width of our chute, and results may differ in significantly wider (or narrower) chutes. The examples shown in Fig. 5 were chosen to be representative of typical flows that we observe in our experiments.

To summarize, in the regular state, we see rapid, noncirculatory flow that accelerates downstream of the sinusoidal crest and slows upstream. In the antiregular state, we see localized high speed flows embedded in a slowly moving mass. We also see *circulatory* flow: this flow is variable, but it seems consistent to surmise that circulation may be generated by strong upstream velocity gradients associated with jets of granular flow over the sinusoidal crest. In the flat flow state, we see nearly uniform velocities that seem consistent with the picture of a sliding layer on a static bed that fills up the sinusoidal troughs. This flow differs qualitatively from the antiregular flow both in asymptotic approach (Fig. 3(a)) and in that flow in the trough is *not* static in the antiregular state. This is evidenced from three sources: from external observations through the transparent side of the chute; from PIV data indicating much slower (and therefore thicker) surface flow above the trough in the antiregular state than in the regular; and from transport determinations, discussed next.

Since grains appear to flow and circulate in the troughs of the antiregular state, we are prompted to raise the obvious question: what is going on in the troughs? In the next section, we explore this issue by examining Lagrangian transport of grains through the trough.

Transport

Grains tend to move slowly and irregularly near the side-walls, and so PIV taken from the side is not revealing; so instead we study complex vertical particle motions by freezing the granular bed after releasing alternating streams of red and blue sand down the sinusoidal chute. We perform these experiments as follows. We position the chute at 31.5° , so as to establish antiregular flow (cf. Fig. 3(a)). We insert a stoppered funnel containing blue sand into the inlet hopper, which we then fill with identical but red sand. This smaller funnel outlet completely blocks the hopper outlet so that no sand flows while the smaller funnel stopper is in place. We initiate flow by pulling the stopper, and when the blue grains are almost exhausted, we rapidly remove the smaller funnel, permitting red grains to flow. We use ample blue grains to fully establish steady antiregular flow, and in this way, we input a uniform flow of grains that abruptly changes from blue to red.

As a first example illustrating the flow dynamics, in Fig. 6(a), we show successive snapshots taken from the side of the chute in the antiregular state. The blue grains have occupied the trough and the red grains are seen to penetrate into the bed from the top as time evolves. In enlarged views (b) and (c), we see

that the mechanism of this penetration is that the bed overturns, subducting the red grains in recirculating loops (indicated by arrows). Two such recirculation loops are evident in Fig. 6(c): a downstream loop dominated by blue grains and outlined by a thin strip of red, and an upstream loop dominated by red grains that encircle a central blue strip of islands. We emphasize that these are not transient effects: the recirculation loops occur in steady, established, flow, and are identifiable only because we have changed the color of the grains in midstream.

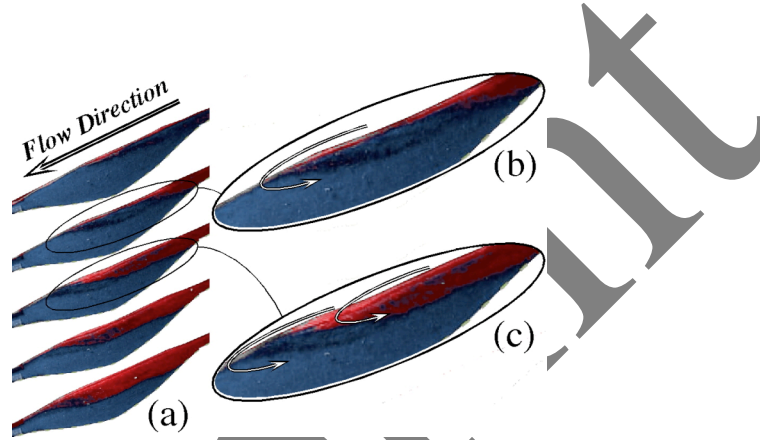


Figure 6. (a) Snapshots of the trough through the transparent side wall of the chute at the 3rd trough from the top during antiregular flow in a two-color sand experiment. The sequence of flow is shown from top to bottom as time progresses. There is an intermittent behavior in the penetration of blue sand into the initially flowing red sand. Enlarged views shown in (b) and (c) suggest subduction of the red sand in multiple recirculation loops, indicated by arrows.

It has been noted in previous work (Shinbrot 1999; Maneval 2004; Taberlet 2003) that granular flows near boundaries often do not faithfully represent interior flows, and so to reveal the transport internal to the bed, we solidify the bed as follows. First, we stop the flow at a time of our choosing by rapidly bringing the chute to a horizontal position. Inflowing (red) sand then spills over the chute near its inlet, and the grains flowing down the chute stop very rapidly – from observation, the grains seem to stop essentially immediately, and by raising the hinged chute smoothly, no backflow or other spurious effects are evident. After stopping the flow, we solidify the (now horizontal) bed by infiltrating it with a low viscosity polymeric solution (Rave®, Unilever, Greenwich, CT), and then letting the polymer set over a period of two weeks (Wightman et al., 1995). The solidified bed of sand is finally removed, cut and labeled. In Fig. 7(b), we show a representative photograph of the resulting cut section.

We remark that the flow patterns produced by this technique are likely not absolutely identical with those that would be seen if the bed could be truly instantaneously frozen in time. Nevertheless, the patterns of flow that we observe seem to faithfully reveal the underlying behaviors that we seek to understand. In particular, as we will describe, patterns of blue and red interfaces develop more detail at successive downstream troughs in a way consistent with the assumption that similar, iterative, circulatory flows (Maneval et al.; Shinbrot et al., 1999; Taberlet et al., 2003) are present in each trough.

Using this solidification technique, in Fig. 7(a) we show a view from the bottom of the bed: this is a snapshot of the solidified mass after it has been lifted off of the sinusoidal supporting plate – the valley of the sine is in the center of the snapshot, and peaks of the sine are at top and bottom. This particular snapshot is taken from the third well from the top, shortly after the red grains have fully extended along the length of the chute. In this view, we can identify two features of interest: first, the darker, blue, grains form multiple striations, suggesting that a repeated sequence of processes have overturned the bed from top (red) to bottom (blue). Second, the striations are significantly curved into ‘surrideos’ (smile-shapes). We indicate these by green broken lines. This is as we would qualitatively expect for a flow that is nearly stalled near the side-walls and flows more rapidly toward the chute centerline. We will simulate the apparent velocity field in the next section, and both of these facts will come into play as we construct the simulation.

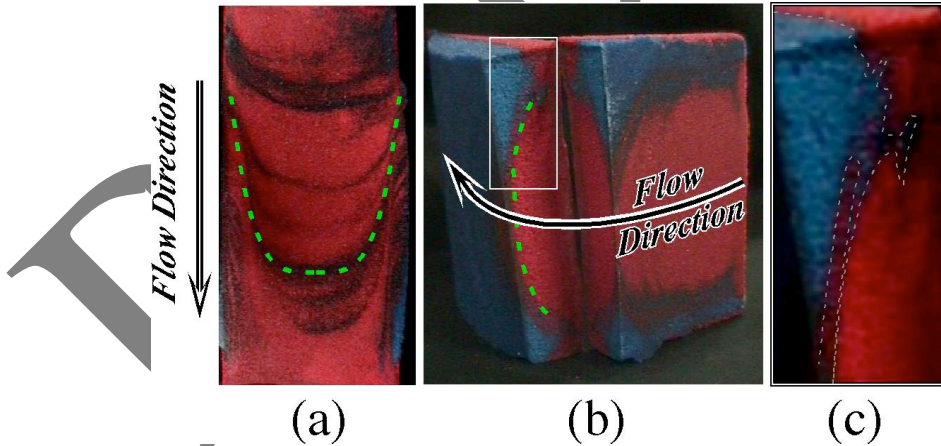


Figure 7. (a) Snapshot from the bottom of a trough after the flow has been stopped, the bed has been frozen (see text), and the solidified bed has been removed from the chute. Curved ‘surrideo’ shapes are evident: the dashed curve is the quartic function used in equations [4]-[5]. (b) Solidified bed removed from the 7th trough downstream. This section of bed has been cut to expose the internal structure. (c) Enlarged view of rectangular section from (b). Photographic resolution is limited by the size of bed grains, so we identify the approximate red/blue interface with broken lines.

In another view in Fig. 7(b), we display the contents of a solidified well (the seventh from the top of the chute) that has been sawed open to reveal the internal structure of the bed. The bottom of the well is

facing the camera, and one side surface is visible at the top of the snapshot. From this snapshot, it appears that a rounded bolus of red grains projects into the bed of blue grains, evidently leaving a curved interface in the cross-stream plane. Here again, the wells further down the chute have more interfacial striations than the wells upstream, suggesting that projections form within each well and are convected downstream. The precise details of the process by which projections infiltrate and mix are clearly complex, as indicated in the enlarged view in Fig. 7(c). Here we see that foliated striations (locations approximated by gray broken lines) appear at the interface between red and blue regions, possibly as a result of intermittent stick-slip motion (Shinbrot et al., 1999).

Velocity Fluctuations:

Granular flows tend to be somewhat noisy, and we did not obtain informative data on velocity fluctuations using short-term measurements. Instead, we have characterized temporal velocity fluctuations by comparing successive moving averages from velocimetry data. For each of the flow regimes, we form differences between successive average flows as follows. Using the surface velocimetry data from Fig. 5, we define the 500-frame (1 second) velocimetry average from frame 1 to frame 500 (within the fourth well from the top of the chute) to be V_1 , and the average from frame 501 to frame 1000 to be V_2 . We then define the sample velocity fluctuation to be $\Delta V = V_1 - V_2$, which we plot as a function of position within the well in Fig. 8. We see no significant temporal fluctuations for regular flow but as shown in Fig. 8, we do see fluctuations for examples of fully established antiregular, and flat flows.

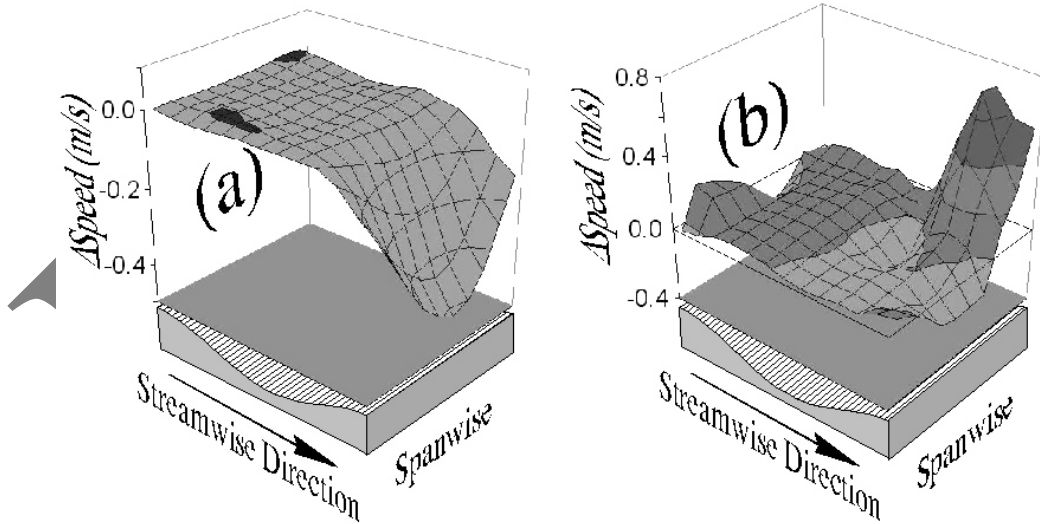


Figure 8 – Sample velocity fluctuations between two successive seconds of PIV acquisition for: (a) flat flow at 30.3° and (b) antiregular flow at 31.4°. The highest peak in (b) correlates with the presence of a transient circulation region (inset to Fig. 5) in this particular case.

During flat flow (Fig. 8(a)), fluctuations remain small and broadly distributed, but in each well, the surface does appear to uniformly slip downstream, slow, then slip again. Thus speed variations in Fig.

8(a) remain small except at the lower section of the trough, where during this slip-slow cycle a slip motion in one well impinges on a slower motion in the next. In antiregular flow (Fig. 8(b)), by contrast, we see much more dynamic variations in velocity (note scale differences in Fig's 8). The large peak in the lower region of the trough for this antiregular case corresponds to local increases in speed that are associated with the circulatory flow highlighted in the inset to Fig. 5.

Visualization:

The experimental data for regular and flat flows reveal few surprises: regular flow is steady and accelerates with the local slope of the chute bottom, hugging its sinusoidal shape. Flat flow by contrast appears to fill in the troughs, and to exhibit a sliding surface layer that slips and slows periodically, but is by and large mundane. Antiregular flow, on the other hand, displays substantial transient accelerations (Fig. 8(b)) including circulatory flows that sometimes emerge at the surface (Fig. 5). Averaged over time, transport produces ‘surrideo’ patterns (Fig. 7) whose cause is not entirely clear-cut. In this section, we seek to better understand how these patterns may be produced. We do this by constructing a Lagrangian flow simulation that permits us to visualize the transport produced by the flow profiles implied by our experimental frozen sections. It is difficult to accurately model granular flows that span kinetics from solid-like to fluid-like (Campbell et al., 1995; Gray and Hutter, 1997), and so instead in this simulation, we *assume* velocity fields that agree with observations and we track markers under the influence of these fields. This is not a first principles simulation: it is merely a visualization method that we use to provide an understanding of how the internal flow seems to behave based on the fixed time snapshots that are attainable.

We generate transport simulations by advecting a plane demarking an imagined interface in the granular bed using the following explicit velocity field, chosen to correspond to data presented in Fig's 6 and 7:

$$V_x = V_0 \left[\frac{1 + 2(1+z)}{4} + V_2 z |z-1| \right] (1 - V_1 y^4) \quad [4]$$

$$V_z = -V_0 (V_2 x |z-1|) (1 - V_1 y^4), \quad [5]$$

In these equations, $V_{0,1,2}$ are constants (we use $V_0 = V_2 = 0.1$; $V_1 = 0.5$ in Fig. 9), and the coordinate system is as shown in Fig. 9(e), where x is in the streamwise direction, z is vertical with $z=1$ at the free surface of the bed. The effects of the separate and combined terms in this velocity field are illustrated in Fig. 9(a)-(f).

In Fig. 9(a)-(b), we show how the first term in the V_x equation deforms an initial plane of markers. This shear velocity field is chosen after quantitative simulations performed elsewhere (Zheng and Hill,

1996; Forterre and Pouliquen, 2003). Similarly in Fig. 9(c), we show how the same initial plane of markers is deformed by the quadratic flow on the right of Eq. [4], and in Fig. 9(d) we show the combined effects of the shear of Fig. 9(b) and the quartic flow of Fig. 9(c). This quartic field is chosen to correspond to the transport data shown in Fig. 7(a): the green dashed line in this figure is such a quartic. The remaining terms in Eq. [4] & [5] define a rotation meant to describe the iterative overturning of the bed seen in Fig. 6.

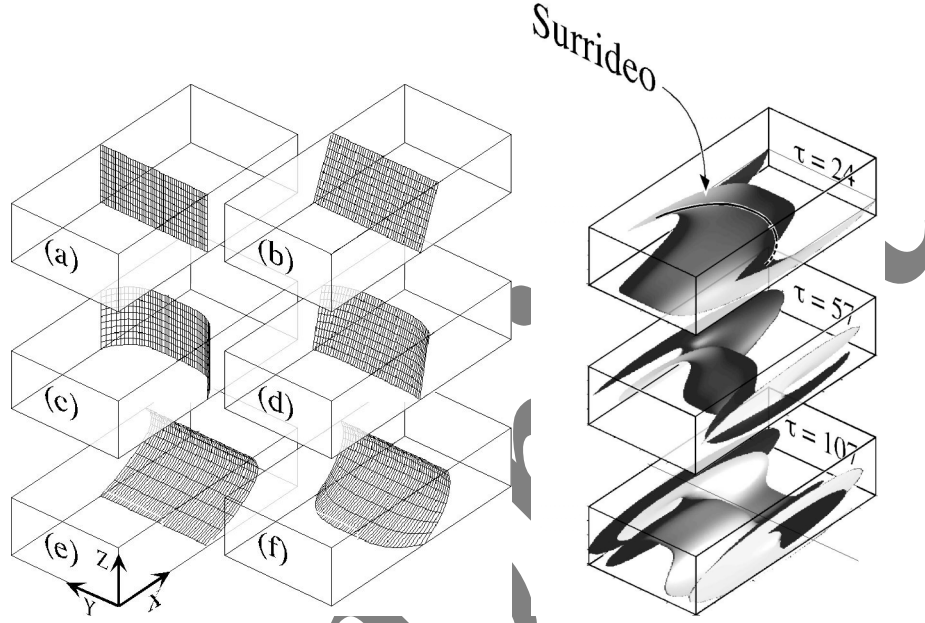


Figure 9 – Simulated Lagrangian flow used to visualize internal particle motion. (a)-(f) illustrations of how an initial plane of markers is transported by the assumed velocity field. (a) Initial plane; (b) First part of assumed velocity: shear produced through drag against the bottom of the chute; (c) Second part of the assumed velocity: a quartic profile shown deforming the initial plane of (a); (d) Combined effects of shear and quartic flow; (e) Combined effects of shear and overturning described in text; (f) Combined effects of all of shear, quartic flow and overturning on the initial plane of (a). At right is the result of advecting the plane of (a) through 24, 57, and 107 timesteps under the combined effects of these flows. The iterated ‘surrideo’ shape where the curved surface intersects with a horizontal plane is identified at $\tau = 24$; every time that the bed turns over, this shape is re-introduced.

To illustrate how these flows would deform an initially vertical interface, at the right of Fig. 9, we show the shape taken by the initial plane of Fig. 9(a) after $\tau = 24, 57$ and 107 timesteps of Lagrangian transport. At $\tau = 24$, we identify the ‘surrideo’ shape that is iteratively repeated at later times (not shown to limit clutter). From these Lagrangian transport simulations, we see that each time the bed overturns under the influence of acceleration into and out of successive troughs, a ‘surrideo’ or smile shape is generated.

Conclusion

Our experiments indicate that the bifurcation identified in the 19th century for fluid flows over a wavy bottom has a qualitative analog in granular chute flows. This analog, however, comes with significant differences in detail. The first of these details is that in the granular case, there are three, rather than only two, distinct states: a flat state at shallow angles of incline, an antiregular state at intermediate angles, and a regular state at the steepest angles. The flat state appears after the trough between successive peaks fills with stalled grains, on which a shallow layer of grains slides nearly uniformly. In this state, kinetic energy appears to be dissipated by slow frictional contacts between the stalled layer beneath and the sliding layer above. The antiregular state by contrast appears from all evidence to be characterized by circulatory flows, which dissipate energy through repeated overturnings of surface layers into the bed. Finally, the regular state exhibits the most rapid mean velocity, with little evidence of mixing, circulation, or enduring sliding contacts.

A second major distinction between the fluid and granular transitions is that the fluid transition appears in a solution for inviscid, irrotational flow. In the fluid case, regular flow appears at high Froude number and is kinetically driven, while antiregular flow appears at lower Froude number and is driven by a reduction in pressure where the fluid travels rapidly over the peak of a bottom waveform, and an increase in Bernoulli pressure as the fluid travels more slowly over the trough. This causes flow streamlines to converge near the peaks and to diverge near the troughs of the bottom waveform.

By contrast, in the granular case, we know of no simple description that would predict the presence of three distinct flow regimes or that would define the locations of transition boundaries as shown in Fig. 4. It is dubious whether the Bernoulli equation would ever apply, most notably since sheared grains dilate rather than contract. Moreover granular flows are strongly dissipative in general, and grains on the wavy bottom are often partially or entirely jammed; additionally we have shown experimental evidence for significant circulatory flows in the granular case. On all of these accounts, our granular flows are not well described by inviscid or irrotational equations. Nevertheless, there appears to be fortuitous agreement between fluid and the granular transitions in flow over a wavy bottom that may give optimism for future hydrodynamic treatments.

Beyond these results of fundamental interest, our results give rise to practical conclusions as well. On the industrial side, our data indicate that either improved bed agitation (in the antiregular state) or complete transport (in the regular state) can potentially be achieved by designing chutes with suitable wavelengths, as informed by Fig. 3(b) and 4(a). Additionally, on the geological side, the results suggest that avalanches can display three predictable and distinct regimes of flow, and that by evaluating (or designing) appropriate spacings between natural or artificial obstacles, it may be possible to determine whether avalanches will jet kinetically downslope (for larger spacings on steeper slopes) or overturn in

dissipative circulatory loops (for closer spacings). We await corroborating studies as the scale, geometry, and materials used are modified.

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